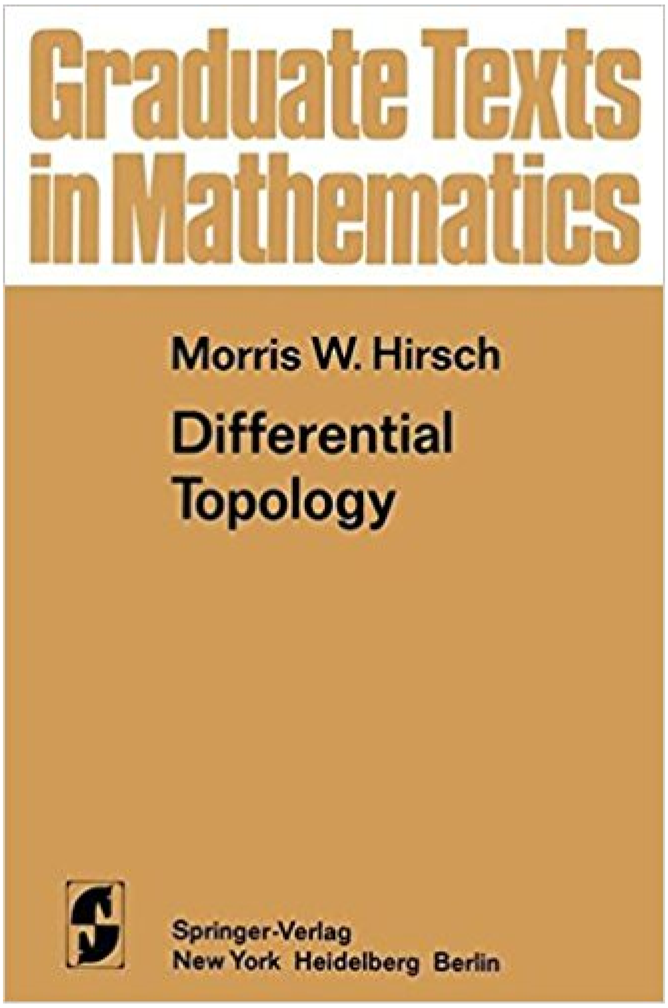
Differential Topology (Graduate Texts In Mathematics)



Synopsis

"A very valuable book. In little over 200 pages, it presents a well-organized and surprisingly comprehensive treatment of most of the basic material in differential topology, as far as is accessible without the methods of algebraic topology....There is an abundance of exercises, which supply many beautiful examples and much interesting additional information, and help the reader to become thoroughly familiar with the material of the main text." Ã¢â ¬â ¢MATHEMATICAL REVIEWS

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Customer Reviews

M.W. Hirsch Differential Topology "A very valuable book. In little over 200 pages, it presents a well-organized and surprisingly comprehensive treatment of most of the basic material in differential topology, as far as is accessible without the methods of algebraic topology. Newly introduced concepts are usually well motivated, and often the historical development of an idea is described. There is an abundance of exercises, which supply many beautiful examples and much interesting additional information, and help the reader to become thoroughly familiar with the material of the main text. "Ã¢â ¬â ¢MATHEMATICAL REVIEWS

Hirsch has assembled a very fine text which is suitable for a second year graduate mathematics course in differentiable manifolds. The development and presentation of the material is quite accessible.The prerequisites or co-requisites for this book are a solid background in general topology. I highly recommend Munkres'Ã Â Topology (2nd Edition). You'll also need a good understanding of analysis as you might find in Rudin'sÃ Â Principles of Mathematical Analysis. It's helpful to know the basics about vector spaces and their duals, and I recommend Hungerford'sÃ Â AlgebraÃ Â for this. Finally, for the material on intersection numbers and Euler characteristic, you might like to know something about homology and cohomology. A good basic reference for this is Munkres'Ã Â Elements of Algebraic Topology.The author begins his study with an introduction of differential manifolds and maps in Chapter 1. Hirsch introduces the tangent space rather intuitively, but his formal definition in terms of equivalences class of triples is not intuitive. This may also seem strange to readers who think of a tangent vector as a first order differential operator. The highlight of the first chapter is a very nice, geometrically motivated prove that smooth compact n-manifolds can be smoothly embedded in 2n+1 dimensional Euclidean space.The main aim of Chapter 2 is to study when a smooth map of manifolds can be approximated by an embedding. This is technical, but crucial material needed for the important study of transversality. To achieve this, the author carefully studies the topology of the spaces of maps between differential manifolds. In order to show that (weakly) closed subspace of maps of manifolds is a Baire space, the concept of an r-jet and the r-jet space are introduced.Chapter 3 is a short chapter which focuses on the all-important concept of transversality and general position. The key to showing that differential maps can be approximated by transverse maps is the Morse-Sard Theorem. The author gives a very nice treatment of this and is carefully only to introduce the simple notion of a set of measure zero and not get side-tracked by a discourse of Lebesgue measure theory.Up next, the notions of tubular neighborhoods and collars are studied by introducing the general concept of a vector bundle in Chapter 4. Basic properties and construction techniques are introduced before covering orientability. The tubular neighborhood is defined and shown to exist. Basic isotopy properties of the tubular neighborhood are studied. The chapter concludes with a study of collars on manifold boundaries.Chapter 5 - 8 are really tantalizing chapters, especially for readers of Rourke and Sanderson'sÃ Â Introduction To Piecewise Linear Topology. Chapter 5 covers some basic algebraic topology concepts such as the degree of a map and intersection numbers. Chapter 6 covers Morse theory and attaching handles. Chapter 7 introduces the notion of a cobordism and Chapter 8 studies the isotopy extension theorem.Equipped with the basic ingredients of the last 4 chapters (plus the Whitney Trick), the reader is now fully equipped to understand Smale's proof of the H-Cobordism Theorem and the proof of the Poincare Conjecture for homotopy spheres of dimensions > 5. This material could easily serve as a springboard into the modern research literature.Unfortunately, Hirsch chose not to pursue the H-Cobordism Theorem and instead gives a handle-theoretic proof of the classification of compact surfaces. This seems like a waste of all of the beautiful theory that has been developed. It is definitely a re-hash if the reader has gone through Munkres'Ã Â Topology (2nd Edition)Ã Â text and studied the very accessible basic proof of surface classification contained there.Each section concludes with a nice set of exercises which often continues the thread of discussion. Exercises have difficulty levels marked by a "*". Two-star problems are important results that are usually published in the research literature. Three-star problems are open, unsolved problems at the time of printing (6th printing, 1997).

This book have so many things of differential topology that are very useful in a lot of branches of mathematics, definitively you should have this book if you are studying math!

I originally bought this book because I wanted to see a more modern treatment of Morse theory, as contained in Milnor's book. Hirsch provides just that, and much more. It's really readable, and doesn't require too much differential geometry to understand.My one comeplaint with this book, and the reason it didn't get 5 stars, is that Hirsch uses a bit too much functional analysis in his book. That's just my taste, and I could be wrong, but I'm not too keen on learning what a "jet" is in the mathematical sense. I don't even really like to fly.The book is very nice in appearence. I'm sure it looks very impressive on my shelf. Many women have commented on it's nice burnt orange color, though it does not match my older springer GTM books.

While I agree with almost everything that reviewers Paul Thurston and Dr. Carlson say about this book, I would rate it a little higher, since this book fills a niche that not too many other books occupy: It is more advanced than truly introductory treatments such as Guillemin & Pollack'sÃ Â Differential TopologyÃ Â or Milnor'sÃ Â Topology from the Differentiable ViewpointÃ Â but more basic than Kosinski'sÃ Â Differential Manifolds, and far more comprehensive than specialized books such as Milnor'sÃ Â Lectures on the h-Cobordism Theorem. Thus it is useful for its coverage of a wide range of topics in differential topology - embeddings, vector bundles, transversality, degree and intersection numbers, cobordism, Morse theory, isotopies - which is rigorous yet still somewhat elementary.Hirsch writes clearly with precise definitions using modern terminology (in contrast to, say, G&P or Milnor's definition of manifold as a subset of Euclidean space). The proofs are usually compact but easy to follow, and he often explains what he is going to do ahead of time. A lot is compressed into relatively few pages - his proof of the equivalence of C^r, r>=1, and smooth structures in Chapter 2 versus Munkres' 60 page proof (inÃ Â Elementary Differential Topology) being a good illustration of this - and results are often proved in much generality.The second chapter in particular stands out, which covers function spaces and approximations and contains a general theorem that immediately yields the denseness of diffeomorphisms, embeddings, immersions, submersions, proper maps, etc., in the strong C^r topology. There is an excursion into jets, which is not found much in the literature, and is not really my cup of tea either, but it is not used much elsewhere in the book. Analytic approximations are also mentioned, although a key result in this area is only cited rather than proved (since this is not a book on complex analysis).Beginning with Chapter 3, transversality is rightly emphasized as a central concept, while Chapter 6 introduces the Morse theory that dominates the last few chapters. A complete proof of the Morse-Sard theorem is given, although only in the smooth case, with a more general theorem only being stated (but still, this is more than any of the other texts I mentioned). Standard results such as the easy Whitney embedding, the existence of collars and tubular neighborhoods, the Brouwer fixed point theorem, the hairy ball theorem, the Hopf theorem, the Morse lemma, and the Morse inequalities are proved in addition to more advanced theorems, such as the classification of vector bundles or Thom's "fundamental theorem of cobordism," although some results, such as the computation of some cobordism groups, are only stated.The chapter on vector bundles is actually the longest in the book, and certainly seems sufficient, although I partially agree with Mr. Thurston that it is odd that the idea of tangent vectors as derivations is not even mentioned. (Tangent vectors are essentially defined as elements of a vector bundle that transform in a certain way under coordinate changes.)It is good to see gluings handled properly without hand-waving about smoothings, and the technical theorems on isotopies and the characterization of the disk from a Morse-theoretic perspective are a welcome addition, but the treatment of cobordism is too brief and handles and surgery are only touched upon. The classification of compact surfaces at the end is kind of overkill with all this machinery as well, but proceeding to handle decompositions or the h-cobordism theorem would make this book much longer. Consider this, instead, as preparation for those topics, which can be learned from, say, Kosinski.Note that this book contains nothing on differential forms, integration, Riemannian geometry, or Lie groups, as it is intended for students of topology itself, rather than those who wish to apply it to study analysis or physics on manifolds. It is certainly not geared toward physicists.With one major type of exception, there aren't that many typos or other errors, but that persistent problem is that frequently the wrong letter is used for a variable, such as f is written for g, or f for F, or U for V, or 1 for 2, etc. Sometimes words or subscripts are omitted, too, and in a few places a cross-reference is given to the wrong theorem number. The proof of Theorem 9.2.1 is a bit sloppy, though, with 3 mathematical typos, an unproven assumption, an unnecessary step, an unmentioned restriction, a couple of paragraphs in reversed order, and a reference to a theorem (8.1.9) that wasn't established in enough generality to be applied here, but no other proof is remotely like this.There are sometimes historical remarks at the ends of sections that contain references to significant extensions. The chapters also begin with relevant quotes from mathematicians, including Whitehead's remark, '"Transversal" is a noun; the adjective is "transverse."' (Someone should tell Lang.) Most sections end with exercises, with many of them being rather challenging. In fact, this book probably has more exercises than any of the other works I cited. Many important results are contained within the exercises, too, although these are not cited elsewhere in the book.